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Risk and the Right Model

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RISK AND THE RIGHT MODEL

All aspects of life in the world are subject to risk and uncertainty. Risk and uncertainty are key characteristics of any long range planning and cost estimation. Few, if any, decisions are made under conditions of certainty and without risk. Due to the complexities involved, analysts and decision makers must specifically and explicitly address this risk and uncertainty in performing their assigned tasks. Although the terms risk and uncertainty are often used interchangeably, they are not the same. Risk is the probability that a planned event will not be attained within constraints (cost, schedule, performance) by following a specified course of action [7:18]. Uncertainty is incomplete knowledge [7:18]. Fisher [3:202] says, "A risky situation is one in which the outcome is subject to an uncontrollable random event stemming from a known probability distribution. An uncertain situation, on the other hand, is characterized by the fact that the probability distribution of the uncontrollable random event is unknown." Canada [1:252] relaxes these definitions somewhat by concluding that risk is the dispersion of the probability distribution of the element under consideration while uncertainty is a lack of confidence that the probability distribution is correct. It is the task of analysts to try to reduce uncertainty to risk and then to meaningfully convey the risk to decision makers.

RISK VERSUS SENSITIVITY ANALYSIS

Analysts cannot eliminate risk and uncertainty from an analysis. At best, they can only present and explain those aspects of risk and uncertainty affecting the decisions. This is done through risk and sensitivity analysis.

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Risk analysis ~~is~~ as a procedure for analyzing how randomness affects the total cost. To place a cost estimate in proper perspective, it must ~~be viewed as a~~ random variable. By definition, a random variable is a numerically valued function defined over the sample space [5:327]. Unfortunately, the application of risk analysis seems limited. Authors and analysts, such as Large [6], McNichols [9], and Worm [10] have, however, addressed the problem of risk in hardware cost estimation.

Uncertainty is addressed through the application of sensitivity analysis. Although often mistakenly used as a substitute for risk analysis, sensitivity analysis is designed to systematically explore the implications of varying assumptions about the future environment and is normally centered on the cost drivers where a range of alternative parameters is investigated. The objective is to identify those parameters whose change will impact the decision at hand. Risk analysis and sensitivity analysis are complementary and, as such, are a vital and necessary part of every cost analysis.

WHY PERFORM RISK ANALYSIS

Normally, decision makers are presented with only a point or 'most likely' cost estimate, with no indication as to the risk (variability) in that estimate. For example, Figure 1 shows the relative cost of two systems, A and B. Using cost as the evaluation criterion, and with all other factors being equal, decision makers would choose System A as it offers the lower cost. But, point estimates can be misleading and can lead to a worse decision than had no estimate at all been used.

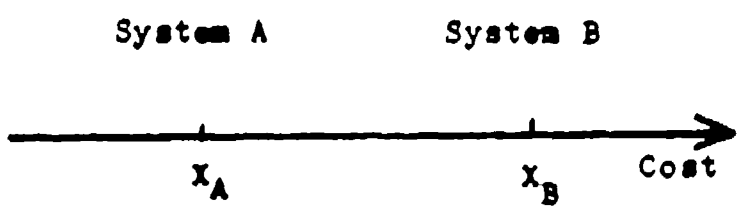

Figure 1 Cost Decision Making

To cite an example from Dienemann [2:2-4], Figure 2 shows four cases in which estimates are expressed as probability distributions to reflect the actual, though perhaps, unmeasurable, risk surrounding each estimate. In Case I, as in Figure 1, decision makers are faced with no real decision problem because all possible costs of System A are lower than System B; using the point estimate would not affect the decision. The situation in Case II is slightly different in that there is some probability that the actual cost of System A will be higher than System B. If this probability is not large, the decision makers would still select A. However, when the overlap is significant, the point estimate would no longer provide a valid datum for system selection. In the third case, both point estimates are the same, but the cost distribution for B has a larger range or variance. Here, decision makers preference toward risk must enter the decision process. If they prefer to minimize risk, they will select A. Case IV is a more complicated situation where the expected cost of System B is lower, but much less certain than A. In this case, if decision makers were to use only a point estiate, they could easily make a wrong or undesirable decision. The application of risk analysis would give much needed visibility into such a decision problem.

Unfortunately, the application and presentation of risk analysis has met with mixed feelings. Many decision makers were only interested in a point estimate [8:42-44]. There are four predominate reasons for this. First, presenting more than a point estimate may constitute an information overload. Cost is but one input to the decision process. Information presented must be clear, concise, and easily understood. This leads to the second reason; some decision makers would not understand risk analysis and its associated implications. Third, the possibility of high costs would cause undue concern and adversely affect the decision. Fourth, risk analysis would impact the credibility of the study giving the impression that analysts were unwilling to stand behind their analyses. Most decision makers do agree, however, that analysts should do risk analysis for their own benefit and in support of the point estimate.

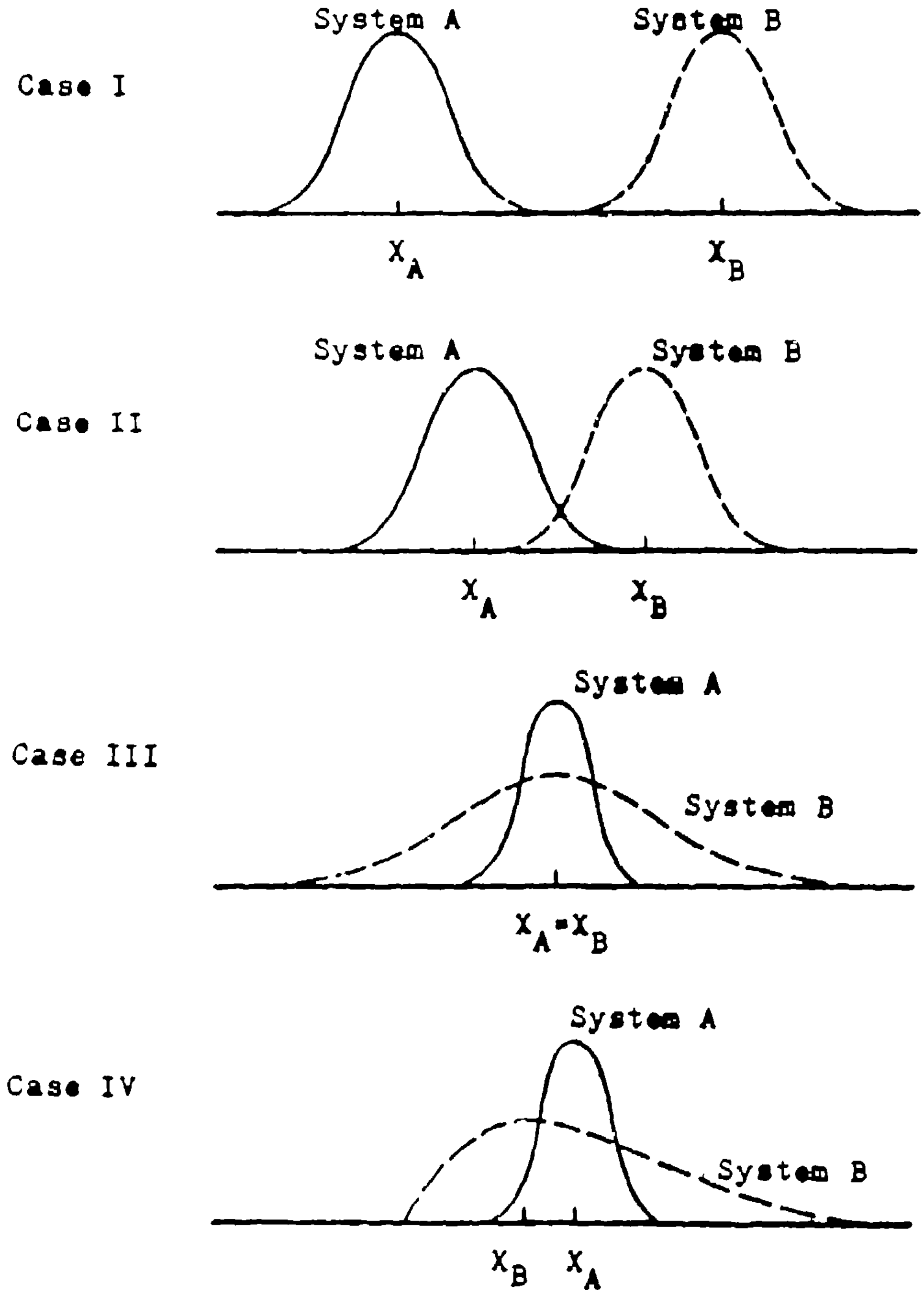

Figure 2 Impact of Cost Risk on Decision Making

But, risk analysis provides precisely the information that the decision makers need. If alternatives cannot be clearly separated and evaluated on the basis of cost, if competing cost estimates fall within the error of the estimate, then the decision should be based on some criterion other than cost. If the probable cost range is too broad, steps should be taken to refine the estimate and decrease the range. Such steps include better data collection and improved estimating methods and techniques. If the possibility of high costs is so significant as to make the system potentially unaffordable, decision makers should be aware of this prior to the decision. Ignoring such information does not lead to better decision making. On the contrary, it leads to cost overruns, unsupportable systems, and impaired readiness.

MODELS AND MODELING

The models or mathematical expressions used in cost analysis appear in two general forms: the additive model and the multiplicative model. The additive model is expressed as

$$Y = X_1 + X_2 + \ldots + X_n \qquad (1)$$

and the multiplicative model as

$$Y = X_1 X_2 \ldots X_n \qquad (2)$$

where X_1 through X_n are random variables. The mathematical expressions used in practice appear to be more complex, but are usually reducible to these two general forms.

From earliest grade school, it is taught that multiplication is a shortcut to addition, and, although this is true, this practice can have unexpected consequences when doing a risk analysis. Substituting multiplication when addition is a more appropriate operation will cause distortion and inaccuracy. Thus, the analyst's choice of a basic model is very important. As an example, consider the two alternative cost estimating relationships.

$$Y = 4X \qquad (3)$$

and

$$Y' = X + X + X + X \qquad (4)$$

Although they may appear to be the same at first glance, they are not. The expected values of equations 3 and 4 respectively are

$$E[Y] = 4E[X] \tag{5}$$

and

$$E[Y'] = E[X]+E[X]+E[X]+E[X] = 4E[X] \tag{6}$$

which reinforces the notion that the models are the same, but the variances of equations 3 and 4 respectively are

$$\mathrm{Var}[Y] = 4^2\mathrm{Var}[X] = 16\mathrm{Var}[X] \tag{7}$$

and

$$\mathrm{Var}[Y'] = \mathrm{Var}[X]+\mathrm{Var}[X]+\mathrm{Var}[X]+\mathrm{Var}[X]$$

$$= 4\mathrm{Var}[X] \tag{8}$$

Upon examination of the variances, it is clear that they are not the same model. The point of this demonstration is to reinforce the principle that models should be a reflection of reality. If the reality of the situation suggests equation 3, then use it. But, if reality suggests equation 4, do not use equation 3 as a shortcut for representing an equation 4 situation.

Next consider the more complicated situation represented in the multiplicative case by the product of two random variables and in the additive case where the number of terms in the sum is itself a random variable. The multiplicative model is

$$Y = NX \tag{9}$$

and the additive model is

$$Y' = X_1 + X_2 + \ldots + X_N \tag{10}$$

where X and N are independent random variables. The expected values of equations (9) and (10) are respectively:

$$E[Y] = E[N]\ E[X] \tag{11}$$

and

$$E[Y'] = E[N]\ E[X] \tag{12}$$

which once again reinforces the notion that the models are the same. The variances are, however, once again a different story. The variances of equations (9) and (10) are respectively

$$Var[Y] = E[N]^2 Var[X] + E[X]^2 Var[N] + Var[N] Var[X] \quad (13)$$

and

$$Var[Y'] = E[N] Var[X] + E[X]^2 Var[N] \quad (14)$$

Once again there is a significant difference in the variances. This difference, as manifested in the multiplicative model, can be quite misleading to the decision maker causing risk analysis to be rejected due to absurdly high variances.

As a practical example, consider the cost of maintenance personnel for a theoretical new fighter, the XF-1. Let X be the pay of an individual maintenance person and N the number of personnel necessary to support a squadron of the new fighter. Expected value and variance of X and N are as shown in Table 1.

TABLE 1

FX-1 Maintenance Personnel

	X	N
E	17,513	1763
Var	23,703,700	35

Letting MP represent total maintenance personnel cost,

$$E[MP] = 1763 \times 17513 = 30,875,419$$

for both the multiplicative and additive models. The variance for the multiplicative model is

$$Var[MP] = 1763^2 \times 23,703,700 + 17513^2 \times 35 + 23,703,700 \times 35$$

$$= 7.3687 \times 10^{13}$$

By the same token, the variance of the additive model is

$$Var[MP] = 1763 \times 23,703,700 + 17,513^2 \times 35$$

$$= 5.2524 \times 10^{10}$$

which is a reduction of three in magnitude. This difference is significant. Tolerance limits with a .95 probability level that 95 percent of the population is included in the limits substantiate this conclusion [8:110-111]. The limits for the multiplicative model are

(14,050,595 , 47,700,243)

and for the additive model are

(30,426,223 , 31,324,615).

Note the difference in the range covered by these two intervals.

CONCLUSION

In many cases, the additive model is the more appropriate model. If individual entities within the population under consideration are exactly the same, then the multiplicative model should be used. But if small differences exist, then the additive model is more justified. To illustrate this point, consider the cost of training pilots. If the cost for each pilot is exactly the same, one would use the multiplicative model and arrive at total cost by multiplying the number of pilots times the cost per pilot. If, however, the cost varies among individual pilots, the additive model is more valid. These cost differences may occur if a particular pilot has to abort a flight due to aircraft system failure and then reaccomplish that flight or if a pilot needs additional flights in order to attain a required level of proficiency.

Use of the additive model is further reinforced by most data collection systems. Data are normally gathered and aggregated additively. Good examples are accounting systems.

Risk analysis is but one more tool in the hands of the decision maker. Unfortunately, all too often this tool is rejected. One reason is the choice of models used by the analyst preparing the estimate. This choice must be a conscious one for the ramifications are great.

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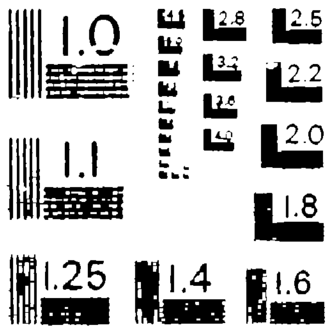

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